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Steadiness of organisms with commensalism using non-linear feedback control

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Abstract

In the present investigation, the local and global steadiness of a two organism's ecological commensalism model is discussed by using non-linear feedback controllers about the interior steady state.

Keywords:

Steadystates;
commensalism;
steadiness;
non-linearfeedbackcntrol.

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1. Introduction

Ecology is the division of biology that deals with the interactions of organisms to one another and to their bodily environments. It is also an interdisciplinary field that includes geography and earth science. There are different types of interactions popularised in ecology studied by several scientists such are named as mutualism, commensalism, parasitism, competition, and predation. Any interaction between organisms and its impact on environment can be evaluated and exposed by using various tools. It can be analysed and by formulating and implementing with an appropriate mathematical model. Mathematical modelling is one of the suitable tool which can used in any complex dynamics. It is one of the most wide ranged tool for many researchers in analysing ecological systems. Many researchers [1], [2], [3], [4] worked on ecological system with various types of interactions using mathematical modelling. Steadiness is one of the major behavioural analysis which is attracted by many researchers. Thus, many

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researchers [5], [6], [7],[8],[9] are studying and analysing the system dynamics and behavioural analysis in various methodologies and techniques. They found the steadiness behaviour analysis by considering both local and global view using various tools where the species are under various interactions. In this article, by formulating a model which includes one of the interaction between species i.e mutualism and studied the dynamics. Even though, many authors studied about this model, our main aim is to expose the global stability behaviouralanalysis by using non-linear control for mutualistic species more innovatively. Our innovative approach in the current model is non- linear control for obtaining global stability.

2. Stability analysis:

A Mathematical model of a two organism's ecological commensalism is given by the following equations.

$$N_1'(t) = N_1\left[a - vN_1 + sN_2\right] \quad (2.1)$$

$$N_2'(t) = N_2\left[n - mN_2\right] \quad (2.2)$$

where N_1, N_2 represents bio mass densities of populations ; a, n are the natural growth rates of populations; v, m represents the rates of decrease of populations; of commensal, host organisms, s is the rate of increase of commensal population. The possible steady states of the system (2.1)-(2.2) are identified as $B_0(0,0)$, $B_1\left(0,\frac{n}{m}\right)$, $B_2\left(\frac{a}{v},0\right)$, $B_3\left(\frac{ma+ns}{vm},\frac{n}{m}\right)$.

(2.3)

Our present investigation is restricted to B_3 only. In this section, it is discussing the steadiness by using the Jacobean matrix of the system (2.1)-(2.2) about B_3. The corresponding characteristic equation is given by

$$\lambda^2 + \left(mN_2 + vN_1 + 2sN_2\right)\lambda + mN_2\left(vN_1 + 2sN_2\right) \quad (2.4)$$

Since $\lambda_1 + \lambda_2 = -\left(mN_2 + vN_1 + 2sN_2\right) < 0; \lambda_1\lambda_2 = mN_2\left(vN_1 + 2sN_2\right) > 0$. Thus B_3 is locally asymptotically stable.

Definition: Bendixson-Dulac criterion:

Let $\dot{x} = f(x,y)$; $\dot{y} = g(x,y)$, where $(x,y) \in R^2$ (2.5)

Let $B(x,y)$ is in C^1 on a simply connected region $D \subset R^2$, if $\frac{\partial(Bf)}{\partial x} + \frac{\partial(Bg)}{\partial y}$ is not identically zero and does not change sign in D, then (2.5) has no closed orbits lying entirely in D.

Theorem (2.1): The interior steady state point B_3 is globally asymptotically stable in the positive quadrant of $N_1 - N_2$ plane.

Proof: Let $H\left(N_1,N_2\right) = \frac{1}{N_1N_2}, N_1 > 0, N_2 > 0$

Let $H\left(N_1,N_2\right)$ is positive in the interior of the positive quadrant of $N_1 - N_2$ plane.

Let $h_1\left(N_1,N_2\right) = N_1\left(a - vN_1 + sN_2\right)$; $h_2\left(N_1,N_2\right) = N_2\left(n - mN_2\right)$

Then $\Delta(N_1,N_2)=\frac{\partial}{\partial N_1}(h_1H)+\frac{\partial}{\partial N_2}(h_2H)=-\frac{v}{N_2}-\frac{m}{N_1}=-\left(\frac{vN_1+mN_2}{N_1N_2}\right)<0$

Clearly $\Delta(N_1,N_2)$ does not change sign and is not identically zero in the positive quadrant of N_1-N_2 plane. Hence by Bendixson-Dulac criterion, there is no closed curve in the interior of R_+^2 of the N_1-N_2 plane. Therefore the steady state B_3 is globally asymptotically stable.

3. Analysis of Non-linear feedback control:

The subject of control of the dynamical system is growing interest of researchers in many different fields such as ecological and environmental modelling, biological and dynamical systems and economics and so on.

Theorem (3.1):

The following nonlinear controllers at the fixed point of the system (2.1)-(2.2) is found to globally asymptotically stable.

$$u_1=vN_1^2-sN_2N_1-2aN_1 \quad (3.1)$$

$$u_2=mN_2^2-2nN_2 \quad (3.2)$$

Proof: A mathematical model of two organism's ecological commensalism having feedback control described given by the equations are as follows

$$N_1'(t)=N_1\left[a-vN_1+sN_2\right]+u_1 \quad (3.3)$$

$$N_2'(t)=N_2\left[n-mN_2\right]+u_2 \quad (3.4)$$

Where N_1,N_2 are state variables and a,v,n,s,m are positive parameters and u_1,u_2 are feedback controllers which are functions of state variable. These control feedback stabilizes the system (3.3)-(3.4) and converges to 0 as $t\to\infty$, i.e, $\lim_{t\to\infty}\|N(t)\|=0$

Now we define candidate Lyapunov function is taken as

$$V(N_1,N_2)=\frac{1}{2}N_1^2+\frac{1}{2}N_2^2 \quad (3.5)$$

Differentiate (3.5) along the trajectories of the system (2.1)-(2.2) gives

$$V'(N_1,N_2)=N_1\left[N_1\left(a-vN_1+sN_2\right)+u_1\right]+N_2\left[N_2\left(n-mN_2\right)+u_2\right] \quad (3.6)$$

Substituting equations (3.1)-(3.2) in (3.6), we get $V'(N_1,N_2)=-aN_1^2-nN_2^2$ which is negative definite function. Thus the two organism's ecological commensalism will be globally asymptotically stable.

4. Numerical simulations:

In this section, we are verifying the steadiness of the system (2.1)-(2.2) using Matlab.

(i) For the parameters $a = 2.5; v = 0.1; s = 1.5; n = 0.5; m = 0.8$

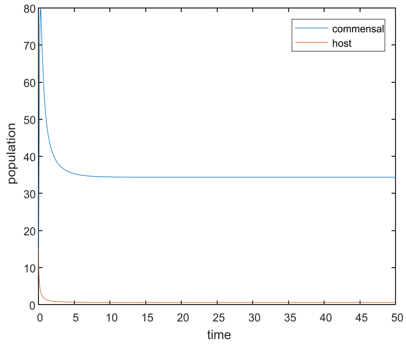

Figure (4.1)

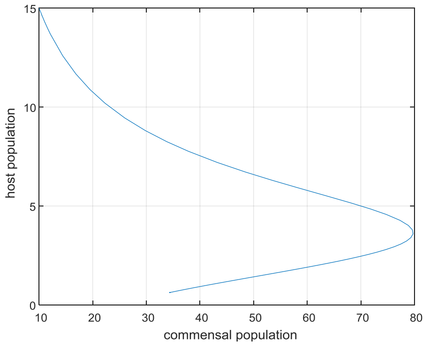

Figure (4.2)

Figure (4.1) shows that the variation of population against time and figure (4.2) shows that phase portrait diagram among commensal and host species

(ii) For the parameters $a = 5.5; v = 0.1; s = 1.5; n = 2.5; m = 0.5$

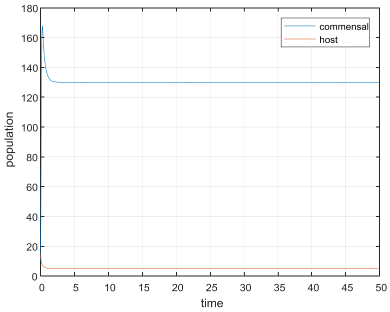

Figure (4.3)

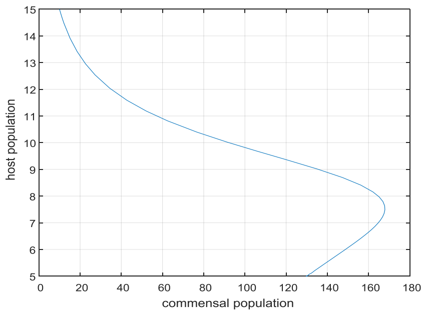

Figure (4.4)

Figure (4.3) shows that the variation of population against time and figure (4.4) shows that phase portrait diagram among commensal and host species

5. Conclusions:

In this, it is premeditated about a commensal–host ideal. We obtained all possible steady states and analysed for steadiness using, non-linear feedback control. Using Bendixson-Dulac criterion, it is also checked the global steadiness of the system (2.1)-(2.2). It is shown that the dynamics of deterministic system in the Figures (4.1)–(4.4) for two different sets of parameters.

References:

[1]. AbdollahGholizadeh, Hasan SaberiNik, Analysis and control of a three dimensional autonomous Chaotic system,
Appl.Math.Inf.Sci., 9(2), 739-747, (2015).

[2]. Al-Ruzaiza, The chaos and control of prey-predator system with some unknown parameters, Applied Mathematical
Sciences, Vol 3,1361-1374, (2009).

[3]. Anderson, R.M. and May, R.M., Population biology of infectious diseases, Part I, Nature, 280, 361-367, (1979).
[4]. Anderson, R.M. and May, R.M., The invasion, persistence and spread of infectious diseases within animal and plant communities, Philos.Trans.R.Soc,Lon, B314, 533-570, (1986).
[5]. Birkoff, G, Rota.G .C, Ordinary differential equation, Ginn, Bosta,1982.
[6]. El-Gohary.A , Al-Ruzaiza. A.S, Optimal Control of Equilibrium State of prey-predator Model, Non-linear dynamics, Psychology and Life Sciences, 6(1), 27-36, (2002).
[7]. El-Gohary.A., Al-Ruzaiza. A.S., Chaos and Adaptive control in two prey, one predator system with nonlinear feedback, Chaos, Solitons& Fractals, 34, 443-453, (2007).
[8]. Hadeler.K.P.,Freedman, H.I., Predator-prey populations with parasitic infection, J.Math.Biol., 27, 609-631, (1989).
[9]. AV S N Murty, M N Srinivas, D R V S R K Sastry, J Venkateswararao, Spatiotemporal, stochastic dynamics and bionomic analysis of an ecological system with harvesting, International Journal of Civil Engineering and Technology, 8(8), 2017, 926–936